EFFECT OF RADIATION ON BOUNDARY-LAYER FILM BOILING

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Heat transfer in film boiling in a forced-convection boundary-layer flow is examined. The interaction between radiation and convection is taken into account by the inclusion of radiation terms in the energy conservation equation. It is found that radiation increases the vapor temperature and the heat conduction on the vapor–liquid interface, whereas heat conduction on the wall is reduced.

Introduction

Film boiling in a forced-convection boundary-layer flow has been investigated by Cess and Sparrow [1], and also by Ito and Nishikawa [2]. In these investigations the radiation of the vapor was neglected. Since radiation is proportional to εT^4, it may be significant in cases where either the emissivity ε_w or the vapor temperature is relatively high. In many engineering applications film boiling takes place at a temperature at which the radiation of the vapor cannot be neglected. The aim of the present work was to investigate film boiling in a forced-convection boundary-layer flow when energy transfer is effected by both convection and radiation.

The physical model chosen for study is the laminar boundary layer over a flat plate (Fig. 1). A stream of liquid impinges on the plate, which has temperature T_w. The liquid is assumed to be saturated. Under the influence of a sufficiently high surface temperature film boiling occurs, i.e., liquid and vapor boundary layers are present. Since the temperatures of the plates and vapor are relatively high energy will be transferred by both radiation and convection. The radiation alters the vapor temperature distribution and, hence, affects the convective heat transfer. This change in convective heat transfer in turn affects the temperature profile and, hence, alters the radiation. Hence, it is obvious that the problem under investigation necessitates a consideration of the combined effect of radiation and convection on the heat transfer.

Analysis

In our investigation we made the following assumptions:

a) the properties of the vapor are constant;

b) the vapor is a nonscattering diffuse absorber and radiator;

c) the plate surface is gray;

d) the product of the vapor absorption coefficient a and the thickness of the vapor film is much less than unity, i.e., $a\delta \ll 1$;

e) the resultant transfer in the x direction within the vapor is negligibly small.

Assumption a) has been made before in many boundary-layer heat-transfer calculations and it has been shown that this assumption is perfectly satisfactory, at least for qualitative results.

If the vapor contains no suspended particles or liquid drops the only type of scattering will be Rayleigh scattering, which is inversely proportional to the fourth power of the wavelength. Generally speaking, the heat radiation wavelengths are so large that such scattering is negligible. On the other hand, scattering may be caused by suspended particles or drops. This type of scattering does not depend on the wavelength

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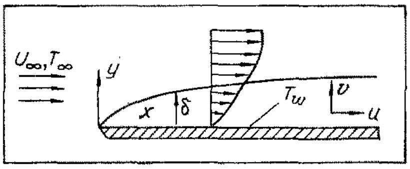

Fig. 1. Physical model and coordinate system.

and will be considerable. Thus, assumption b), i.e., that the vapor is nonscattering, implies the absence of solid particles or liquid drops. Assumption d) implies an optically thin approximation. This assumption is valid in most boundary-layer problems. The next step is to establish the condition in which radiative transfer in the x direction is negligibly small (assumption e). This necessitates, firstly, the formulation of the energy equation for this boundary-layer problem

$$\rho_v c_{p_v}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=-\operatorname{div}\bar{q}_c-\operatorname{div}\bar{q}_r, \tag{1}$$

where the stroke on top denotes a vector quantity and $\bar{q}_c = -k \operatorname{grad} T$. It is well known that if the Peclet number ($Pe = U_\infty x/\alpha$) is sufficiently large, the conductivity in the x direction will be negligibly small, so that $\operatorname{div} q_c \approx \partial q_{cy}/\partial y$, where the subscripts x and y denote the corresponding vector components. Equation (1) can then be written in the form

$$\rho_v c_{p_v}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_v\frac{\partial^2 T}{\partial y^2}-\operatorname{div}\bar{q}_r. \tag{2}$$

The criterion for neglect of radiation can now be determined. This is done by investigating the conditions in which radiation in the direction of the flow becomes negligibly small in comparison with convection, i.e.,

$$\rho_v c_{p_v} u\frac{\partial T}{\partial x}\gg\frac{\partial q_{rx}}{\partial x}. \tag{3}$$

The order of magnitude of $u\,\partial T/\partial x$ is determined in the usual way

$$u\frac{\partial T}{\partial x}\sim U_\infty\frac{T_w-T_{\text{sat}}}{x}. \tag{4}$$

For the evaluation of $\partial q_{rx}/\partial x$ it is essential to note [3] that the optically thick approximation will take into account the radiative heat transfer when this approximation is applied to the case of zero optical thickness. Thus, in the given situation the corresponding criterion will be introduced by using the optically thick approximation. The order of magnitude of $\partial q_{rx}/\partial x$ can then be evaluated from the Rosseland approximation for a radiative heat flux

$$q_r=-\frac{4}{3a}\frac{de}{dx},$$

where e is the emissivity of a blackbody, $e = \sigma T^4$.

Thus,

$$q_{rx}=-\frac{16\sigma T^3}{3a}\frac{\partial T}{\partial x},$$

i.e.,

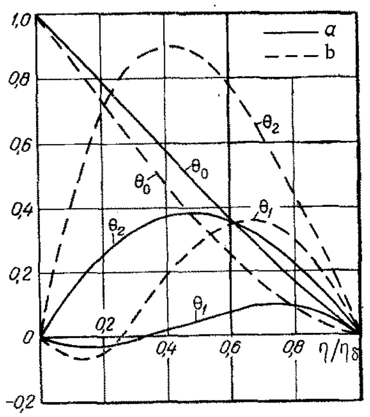

Fig. 2. Temperature profiles for $\eta_\delta = 1.0$ (a) and 2.0 (b).

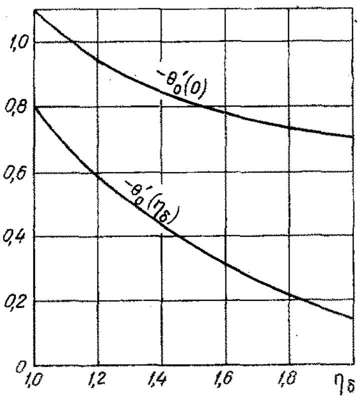

Fig. 3. Graphs of functions $\Theta_0'(0)$ and $\Theta_0'(\eta_\delta)$.

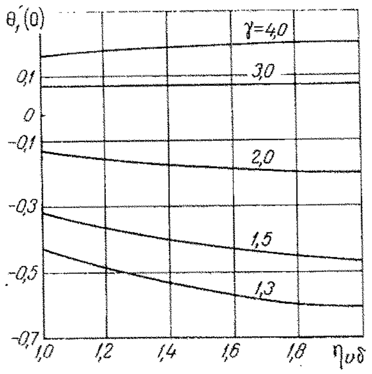

Fig. 4. Graph of function $\Theta_1'(0)$.

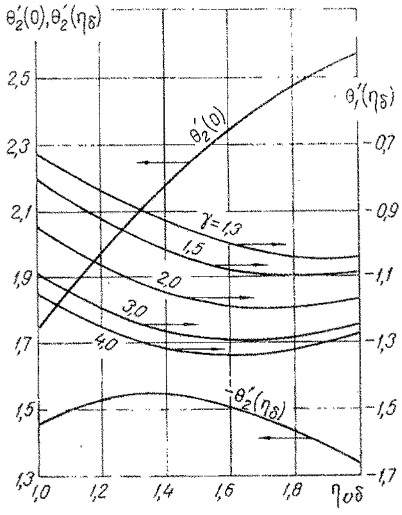

Fig. 5. Graphs of functions $\Theta_1'(\eta_\delta)$, $\Theta_2'(0)$, and $\Theta_2'(\eta\delta)$.

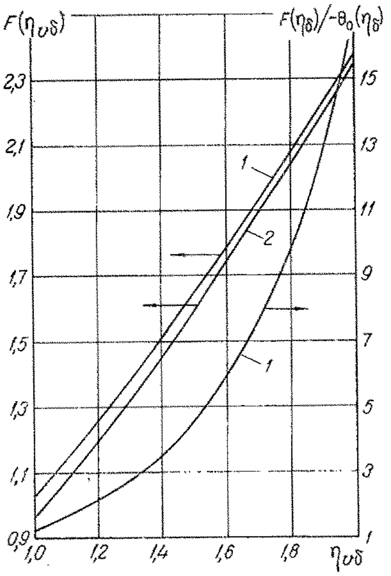

Fig. 6. Graphs of functions $F(\eta_{v\delta})$ and $F(\eta_{v\delta})/[-\Theta_0'(\eta\delta)]$: 1) ratio $(\rho\mu) = 0.01$; 2) 0.1.

$$\frac{\partial q_{rx}}{\partial x} \sim \frac{16\,\sigma T^3}{3a}\,\frac{T_w - T_{sat}}{x^2}. \tag{5}$$

The condition for negligible radiative transfer in the x direction is obtained from equations (3), (4), and (5)

$$\frac{U_\infty x}{16\,\sigma T^3/3a\rho_v c_{p_v}} \gg 1. \tag{6}$$

Radiation of Vapor. Referring to [4] we can express the monochromatic radiation flux in the form

$$q_{r\lambda} = 2\varepsilon_w e_{w\lambda} E_3(\tau_\lambda) + 4(1-\varepsilon_w) E_3(\tau_\lambda) \int_0^{\tau_{\delta\lambda}} e_\lambda(x,\ \tau_\lambda) E_2(\tau_\lambda)\, d\tau_\lambda$$
$$+ 2\int_0^{\tau_\lambda} e_\lambda(x,\ t) E_2(\tau_\lambda - t)\, dt - 2\int_{\tau_\lambda}^{\tau_{\delta\lambda}} e_\lambda(x,\ t) E_2(t-\tau_\lambda)\, dt, \tag{7}$$

where $\tau_\lambda = \int_0^y a_\lambda dy$ is the monochromatic optical thickness; $E_\eta(t) = \int_0^1 \mu^{\eta-2}\exp(-t/\mu)d\mu$ is an exponential integral. Corresponding to this,

$$-\frac{\partial q_{r\lambda}}{\partial \tau_\lambda} = 2\varepsilon_w e_{w\lambda} E_2(\tau_\lambda) + 4(1-\varepsilon_w) E_2(\tau_\lambda) \int_0^{\tau_{\delta\lambda}} e_\lambda(x,\ \tau_\lambda) E_2(\tau_\lambda)\, d\tau_\lambda$$
$$+ 2\int_0^{\tau_{\delta\lambda}} e_\lambda(x,\ t) E_1(|\tau_\lambda - t|)\, dt - 4e_\lambda(x\tau_\lambda). \tag{8}$$

Having $e_\lambda = e_{\infty\lambda}$ for $\tau_\lambda \geq \tau_{\delta\lambda}$ it can be shown that when $\tau_{\delta\lambda} \ll 1$ equation (8) for $\tau_\lambda \ll \tau_{\delta\lambda}$ (i.e., in the boundary layer) reduces to the equation

$$-\frac{\partial q_{r\lambda}'}{\partial \tau_\lambda} = q\varepsilon_w(e_{w\lambda} - e_{\infty\lambda}) + 4(e_{\infty\lambda} - e_\lambda). \tag{9}$$

Thus, we have

$$\frac{\partial q_{r\lambda}}{\partial y} = -a_\lambda\left[2\varepsilon_w(e_{w\lambda} - e_{\infty\lambda}) + 4(e_{\infty\lambda} - e_\lambda)\right]. \tag{10}$$

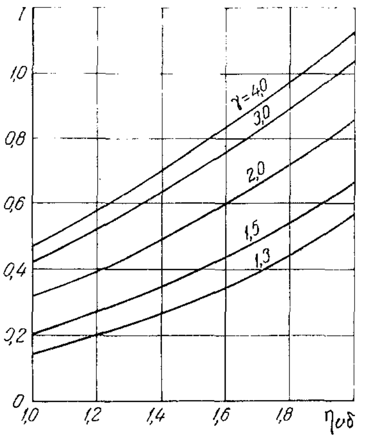

Fig. 7. Graph of function I $= \int_0^{\eta_\delta} H_0(\eta_v) d\eta_\delta$.

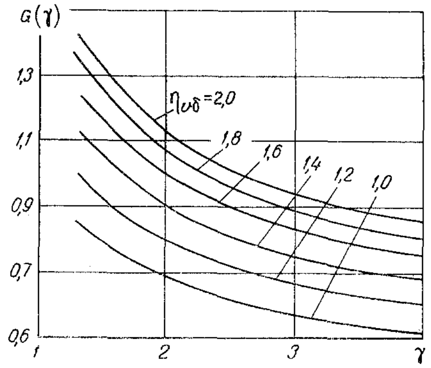

Fig. 8. Graph of function G(γ) (formula (41)).

Boundary Layer Equation

The main differential equations for the present problem can be written in the form:

continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{11}$$

momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu_v \frac{\partial^2 u}{\partial y^2}, \tag{12}$$

energy equation

$$\rho_v c_{pv}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_v \frac{\partial^2 T}{\partial y^2} + \int_0^{\tau_{\delta\lambda}} 2a_\lambda [\varepsilon_w (e_{w\lambda} - e_{\infty\lambda}) + 2(e_{\infty\lambda} - e_\lambda)] d\lambda. \tag{13}$$

For the liquid layer we have only the equations of mass and momentum conservation, since the liquid temperature in the present investigation is assumed to be constant:

continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{14}$$

momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu_L \frac{\partial^2 u}{\partial y^2}. \tag{15}$$

Boundary Conditions

To complete the statement of the problem we formulate the boundary conditions. On the plate surface, owing to the adherence of the viscous liquid, the two velocity components will be zero. In addition, the vapor directly adhering to the wall will have temperature T_W. On the liquid–vapor interface ($y = \delta$) the vapor will have temperature T_{sat}. In addition, on the interface the following conditions will be satisfied:

1) equality of vapor and liquid velocities;

2) continuity of shear stress;

3) conservation of mass across interface.

The velocity at points in the free stream far from the interface will be equal to the velocity of the impinging flow: i.e., for $y = 0$ (vapor phase)

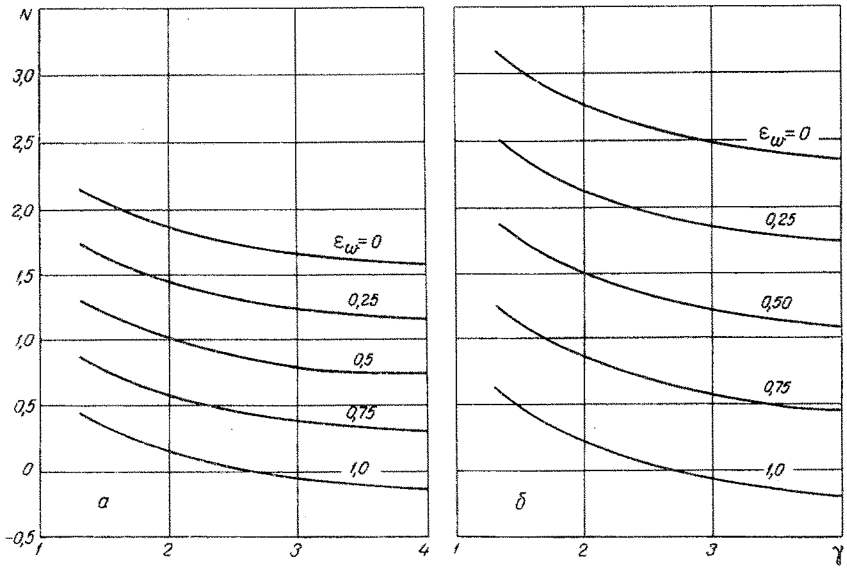

Fig. 9. Convective mass transfer $N = [(\gamma-1)/(\gamma^4-1)]H(\gamma, \varepsilon_W)$ as function of γ: a) $\eta_{v\delta} = 2.0$; b) 1.0).

$$u = v = 0, \quad T = T_w, \tag{16}$$

for $y = \delta$ (liquid–vapor interface)

$$u_v = u_L, \quad \left(\mu \frac{\partial u}{\partial y}\right)_v = \left(\mu \frac{\partial u}{\partial y}\right)_L,$$
$$\rho_v \left(u \frac{d\delta}{dx} - v\right)_v = \rho_L \left(u \frac{d\delta}{dx} - v\right)_L, \quad T = T_{\text{sat}}, \tag{17}$$

for $y = \infty$ (free stream)

$$v \to U_\infty. \tag{16a}$$

These initial and boundary conditions in conjunction with the conservation equations constitute the complete statement of the problem.

We now convert the main equations to more convenient forms and find their solutions.

Gray Vapor

We consider the preceding equations for the case of a gray vapor, i.e., a vapor for which the absorption coefficient $a\lambda$ is independent of the wavelength ($a_\lambda = a$). On this assumption equation (13) has the form

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_v}{\rho_v c_{p_v}} \frac{\partial^2 T}{\partial y^2} + \frac{2\sigma a}{\rho_v c_{p_v}} [\varepsilon_w (T_w^4 - T_{\text{sat}}^4) + 2 (T_{\text{sat}}^4 - T^4)]. \tag{18}$$

The continuity equation can be satisfied by the introduction of a stream function ψ

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}.$$

We introduce the dimensionless variables:

Vapor layer

$$\eta_v = \frac{y}{2} \sqrt{\frac{U_\infty}{\nu_v x}}, \quad F(\eta_v) = \frac{\psi_v}{\sqrt{\nu_v U_\infty x}},$$
$$\gamma = T_w / T_{\text{sat}}, \quad \xi = \frac{2\sigma a T_{\text{sat}}^3 x}{\rho_v c_{p_v} U_\infty}; \tag{19a}$$

liquid layer

TABLE 1. Values of $F(\eta_{v\delta})$

$\eta_{v\delta}$	Ratio $(\rho\mu)$		
	0,001	0,01	0,1
1,0	1,0310	1,0245	0,6945
1,2	1,2658	1,2598	1,2049
1,4	1,5168	1,5115	1,4631
1,6	1,7868	1,7824	1,7415
1,8	2,0781	2,0745	2,0418
2,0	2,3920	2,3893	2,3650

TABLE 2. Temperature Gradients (ratio $[(\rho\mu)_v/(\rho\mu)_L]^{1/2} = 0.01$, Pr = 1)

$\eta_{v\delta}$	$-\Theta_0'(0)$	$-\Theta_0'(\eta_{v\delta})$	$\Theta_2'(0)$	$-\Theta_2'(\eta_{v\delta})$
1,0	1,1016	1,797	1,7262	1,4409
1,2	0,9505	0,5874	1,9689	1,5212
1,4	0,8494	0,4303	2,171	1,5347
1,6	0,7803	0,3089	2,3344	1,4955
1,8	0,7329	0,2146	2,4625	1,4200
2,0	0,7010	0,1424	2,5610	1,3240

$$\eta_L = \frac{y}{2}\sqrt{\frac{U_\infty}{\nu_L x}}, \quad f(\eta_L) = \frac{\psi_L}{\sqrt{\nu_L U_\infty x}}. \tag{19b}$$

The velocity components in the vapor layer then have the form

$$u = \frac{U_\infty F'}{2} \text{ and } v = \frac{1}{2}\sqrt{\frac{\nu_v U_\infty}{x}}(\eta_v F' - F), \tag{20a}$$

and the velocity components in the liquid layer are

$$u = \frac{U_\infty f'}{2} \text{ and } v = \frac{1}{2}\sqrt{\frac{\nu_L U_\infty}{x}}(\eta_L f' - f). \tag{20b}$$

Using the introduced variables in the momentum equations (12) and (15) we obtain

$$F''' + FF'' = 0, \tag{21}$$

$$f''' + ff'' = 0. \tag{22}$$

Equations (21) and (22) are Blasius-type equations for dynamic problems in vapor and liquid, respectively.

The initial and boundary conditions can also be expressed in terms of the new variables. Firstly, it should be noted that the value of η_v on the interface is denoted by $\eta_{v\delta}$. The value of η_v can also be denoted by $\eta_{L\delta}$. Since, however, the actual value of η_L is not contained in the main equations – neither the differential equation (22) nor the boundary conditions – we can, without loss of generality, assume $\eta_L = 0$ at the liquid–vapor interface. Then, using (19), we write condition (16) and (17) in the form:

for $\eta_v = 0$ (plate surface) $F = F' = 0$.

for $\eta_v = \eta_{v\delta}$ (interface) $f_i = [(\rho\mu)_v/(\rho\mu)_L]^{1/2} F_i$,

$$f_i' = F_i', \qquad f_i'' = \left[\frac{(\rho\mu)_v}{(\rho\mu)_L}\right]^{1/2} F_i'', \tag{23}$$

for $\eta_L = \infty$ (free stream) $f' \to 2$.

We will seek the solution of the energy equation (18) in the form

$$\Theta = \frac{T}{T_{sat}} = [1 + (\gamma - 1)\Theta_0(\eta_v)] + (\gamma^4 - 1)[\Theta_1(\eta_v) + (\varepsilon_w - 1)\Theta_2(\eta_v)]\xi + \dots \tag{24}$$

Substituting (24) and (20) in equation (18) and collecting terms with the same powers of ξ, we obtain the usual differential equations for Θ_0, Θ_1, and Θ_2:

$$\frac{1}{\Pr}\Theta_0'' + F\Theta_0' = 0, \tag{25}$$

$$\frac{1}{4\Pr}\Theta_1'' + \frac{1}{4}F\Theta_1' - \frac{1}{2}F'\Theta_1 = -H_0(\eta_v), \tag{26}$$

$$\frac{1}{4\Pr}\Theta_2'' + \frac{1}{4}F\Theta_2' - \frac{1}{2}F'\Theta_2 = -1, \tag{27}$$

where

TABLE 3. $\Pr = 1;\ [(\rho\mu)_V/(\rho\mu)_L]^{1/2} = 1.01$

$\eta_{v\delta}$	T_w/T_{sat}	$\Theta_1'(0)$	$-\Theta_1'(\eta_{v\delta})$	I
1,0	1,3	—0,4331	0,7276	0,1492
	1,5	—0,3277	0,8140	0,2115
	2,0	—0,1399	0,9577	0,3189
	3,0	+0,057	1,093	0,4262
	4,0	0,1531	1,152	0,4761
1,2	1,3	—0,4925	0,8448	0,2029
	1,5	—0,3735	0,9306	0,2766
	2,0	—0,1616	1,073	0,4037
	3,0	+0,0607	1,2053	0,5303
	4,0	0,1689	1,2625	0,5889
1,4	1,3	—0,5402	0,9399	0,2700
	1,5	—0,4105	1,02	0,3546
	2,0	—0,1796	1,1518	0,5000
	3,0	+0,062	1,274	0,6444
	4,0	0,1802	1,3255	0,7112
1,6	1,3	—0,5763	1,0072	0,3531
	1,5	—0,4386	1,08	0,4475
	2,0	—0,1935	1,194	0,6096
	3,0	+0,063	1,2997	0,7701
	4,0	0,1882	1,3444	0,8441
1,8	1,3	—0,6016	1,043	0,4540
	1,5	—0,4582	1,1027	0,5571
	2,0	—0,2032	1,2004	0,7337
	3,0	+0,064	1,289	0,0081
	4,0	0,1940	1,3262	0,9882
2,0	1,3	—0,6175	1,045	0,5736
	1,5	—0,4704	1,095	0,6840
	2,0	—0,2089	1,1757	0,8728
	3,0	+0,0649	1,251	1,0601
	4,0	0,1980	1,2781	1,1345

$$H_0(\eta_v) + \frac{1}{\gamma^{4-1}}\,[1 + \gamma^4 - 2\{1 + (\gamma - 1)\Theta_0\}^4]. \tag{28}$$

The boundary conditions derived from (16), (17), and (24) have the form

$$\begin{gathered}\Theta_0(0) = 1,\ \Theta_1(0) = 0,\ \Theta_2(0) = 0,\\ \Theta_0(\eta_{v\delta}) = 0,\ \Theta_1(\eta_{v\delta}) = 0,\ \Theta_2(\eta_{v\delta}) = 0.\end{gathered} \tag{29}$$

As was to be expected [1, 2], the function $\Theta_0(\eta_V)$ is the temperature distribution for the case of negligible radiation interaction ($\xi = 0$) [1, 2], although the term in the brackets in equation (24) denotes a first-order radiation effect on the temperature profile within the vapor.

$\eta_{V\delta}$ and Physical Properties

The dimensionless thickness $\eta_{V\delta}$ of the vapor film can be related to the known physical properties of the system by writing the energy balance equation on the interface

$$\dot{m}h_{fg} = -k_v\left(\frac{\partial T}{\partial y}\right)_\delta + R. \tag{30}$$

Equation (30) shows that the sum of the local heat conduction and the resultant radiation R on the interface is balanced by the heat of vaporization. In the new variables, equation (30) is written in the form

$$\frac{1}{\gamma - 1}\,\frac{c_p\Delta T}{h_{fg}\Pr} = \frac{1}{-\Theta'(\eta_{v\delta}, \xi)}\left[F(\eta_{v\delta}) - \frac{R}{\rho_v\sqrt{\nu_0}\,h_{fg}}\sqrt{\frac{x}{U_\infty}}\right], \tag{31}$$

where

$$\Theta'(\eta_{v\delta},\ \xi) = [(\gamma - 1)\Theta_0'(\eta_{v\delta})] + (\gamma^4 - 1)[\Theta'(\eta_{v\delta}) + (\varepsilon_w - 1)\Theta_2'(\eta_{v\delta})]\,\xi + \cdot \tag{32}$$

Since $c_p\Delta T/h_{fg}\Pr$ is independent of x, it follows from this that the value of the right side of equation (31) will also be independent of x. Hence, it can be determined for x = L. The resultant radiation flux can be found from the following equation:

$$R = \varepsilon_w\sigma T_{sat}^4(\gamma^4 - 1) + 2\sigma a T_{sat}^4 \int_0^{\eta_{v\delta}} \Theta^4 d\eta_v 2\sqrt{\frac{\nu_v L}{U_\infty}}. \tag{33}$$

Since the radiative transfer between the plate and interface makes the main contribution to R, then we can, for convenience, replace Θ in equation (33) by $[1 + (\gamma-1)\Theta_0(\eta_v)]$ without introducing any appreciable error. Hence, equation (33) can be rewritten as

$$R = \varepsilon_w \sigma T_{\text{sat}}^4 (\gamma^4 - 1) + 2\sigma a T_{\text{sat}}^4 (\gamma^4 - 1) \sqrt{\frac{\varepsilon_v L}{U_\infty}} \left[\eta_{v\delta} - \int_0^{\eta_{v\delta}} H_0(\eta_v)\, d\eta_v\right]. \tag{34}$$

Equation (32) can be written in the form

$$\frac{c_p \Delta T}{h_{fg} \mathrm{Pr}} = \frac{(\gamma - 1)}{-\Theta'(\eta_{v\delta}, \xi_l)} [F(\eta_{v\delta}) - 2C_1 - C_2 - 2C_3], \tag{35}$$

where

$$\Theta'(\eta_{v\delta}, \xi_l) = (\gamma - 1)\Theta_0'(\eta_{v\delta}) + (\gamma^4 - 1)[\Theta_1'(\eta_{v\delta}) + (\varepsilon_w - 1)\Theta_2'(\eta_{v\delta})]\xi_l + \ldots;$$

$$C_1 = \frac{(\gamma^4 - 1) T_{\text{sat}}^4 \varepsilon_w \sigma}{\rho_v h_{fg}} \sqrt{\frac{L}{U_\infty \nu_v}};$$

$$C_2 = (\gamma^4 - 1)\xi_l \frac{c_{pv} T_{\text{sat}}}{h_{fg}} \left[\eta_{v\delta} - \int_0^{\eta_{v\delta}} H_0(\eta_v)\, d\eta_v\right]; \tag{36}$$

$$C_3 = 2\xi_l \frac{c_{pv} T_{\text{sat}}}{h_{fg}} \eta_{v\delta}; \quad I = \int_0^{\eta_{v\delta}} H_0(\eta_v)\, d\eta_v.$$

In the deduction of equation (35) it was assumed that the vapor–liquid interface behaves like a blackbody. However, for a nonblack vapor–liquid interface we have

$$C_1 = \frac{\gamma^4 - 1}{\dfrac{1}{\varepsilon_w} + \dfrac{1}{\varepsilon_L} - 1} \frac{\sigma T_{\text{sat}}^4}{\rho_v h_{fg}} \sqrt{\frac{L}{U_\infty \nu_v}}, \tag{36a}$$

where ε_W and ε_L are the emissivities of the vapor and vapor–liquid interface, respectively. For negligibly small radiation of the interface and vapor ($a = \varepsilon_W = \varepsilon_L = 0$) equation (35) reduces to

$$\frac{c_p \Delta T}{h_{fg} \mathrm{Pr}} = \left[\frac{F(\eta_{v\delta})}{-\Theta_0'(\eta_{v\delta})}\right]. \tag{36b}$$

Equation (35) expresses the relation between the physical properties and thickness $\eta_{v\delta}$ of the vapor film, since $F(\eta_{v\delta})$ and $\Theta'(\eta_{v\delta}, \xi_1)$ are functions of $\eta_{v\delta}$. Hence, the vapor film thickness $\eta_{v\delta}$ can be found for specified values of the physical properties.

Results of Heat Transfer Investigation

For evaluation of the heat transfer from the plate surface, it is convenient to consider separately the radiative heat transfer and the conductive heat transfer. The radiation from the plate consists of two components – radiation from the wall to the liquid and radiation from the wall to the vapor:

$$q = q_c + q_{wr}, \tag{37}$$

where

$$q_c = -k_v \frac{\Delta T}{(\gamma - 1)} \frac{1}{2} \sqrt{\frac{U_\infty}{\nu_v x}} \Theta'(0);$$

$$\Theta'(0) = (\gamma - 1)\Theta_0'(0) + (\gamma^4 - 1)[\Theta_1'(0) + (\varepsilon_w - 1)\Theta_2'(0)]\xi_l. \tag{38}$$

To determine the total radiative heat transfer from the plate surface we consider equation (7) when $\tau_\lambda = 0$

$$q_{wr} = \varepsilon_w \int_0^\infty \left[(e_{w\lambda} - e_{\infty\lambda}) - 2\int_0^\infty (e_\lambda - e_{\infty\lambda}) E_2(\tau_\lambda)\, d\tau_\lambda\right] d\lambda.$$

As before, within the limits of the first approximation adopted in the present analysis we can put $e_\lambda = e_{\infty\lambda}$ for $\tau_\lambda \geq \tau_{\delta\lambda}$, whereas for $\tau_\lambda < \tau_{\delta\lambda}$ we have $E_2(\tau_\lambda) = 1$. Thus, the equation written above will take the form

$$q_{wr} = \varepsilon_w \int_0^\infty \left[(e_{w\lambda} - e_{\infty\lambda}) - 2\int_0^\infty (e_\lambda - e_{\infty\lambda})\, d\tau_\lambda\right] d\lambda. \tag{39}$$

For a gray vapor the absorption coefficient is independent of wavelength ($a_\lambda = a$) and equation (39) reduces to the following:

$$q_{wr} = \varepsilon_w (e_w - e_\infty) - 2\varepsilon_w \int_0^\infty (e - e_\infty)\, d\tau. \tag{40}$$

When (24) is substituted in this equation and terms of the order $\zeta\xi$ are neglected we obtain

$$\frac{q_{wr}}{\varepsilon_w (e_w - e_\infty)} = 1 - G(\gamma)\,\zeta + \ldots, \tag{41}$$

where

$$G(\gamma) = \frac{2}{\gamma^4 - 1}\int_0^{\eta_{v\delta}} [\{1 + (\gamma + 1)\Theta_0\}^4 - 1]\, d\eta_v;$$
$$G(\gamma) = \int_0^{\eta_{v\delta}} \{1 - H_0(\eta_v)\}\, d\eta_v \tag{42}$$

and ζ is a measure of the optical thickness of the boundary layer, given by

$$\zeta = \frac{ax}{\sqrt{\mathrm{Re}}} \sim \tau_\delta. \tag{43}$$

Hence, the total heat transfer on the wall is given by the expression

$$q = -\frac{k_v}{2}\frac{\Delta T}{(\gamma - 1)}\sqrt{\frac{U_\infty}{\nu_v x}}\,\Theta'(0) + \varepsilon_w \sigma T_{\mathrm{sat}}^4 (\gamma^4 - 1)\left[1 - \frac{ax}{\sqrt{\mathrm{Re}}}\left\{\eta_{v\delta} - \int_0^{\eta_{v\delta}} H_0(\eta_v)\, d\eta_v\right\}\right]. \tag{44}$$

It should be noted that if the vapor neither absorbs nor emits, the resultant radiation from the plate is simply

$$q_{wr} = \varepsilon_w \sigma T_{\mathrm{sat}}^4 (\gamma^4 - 1). \tag{45}$$

Hence, the second term in the brackets on the right side of equation (44) is a first-order correction, due to the presence of vapor, to the expression written above. It should be noted that this first-order correction depends only on the optical thickness ζ and the temperature ratio γ and is independent of the plate emissivity ε_W and the expansion parameter ξ.

If we define the Nusselt number as

$$\mathrm{Nu} = \frac{q_c x}{k_v (T_w - T_{\mathrm{sat}})} = -\frac{x}{\Delta T}\left(\frac{\partial T}{\partial y}\right)_w, \tag{46}$$

then equation (38) can be converted, for convenience, to the form

$$\frac{\mathrm{Nu}}{2\sqrt{\mathrm{Re}}} = -\Theta_0(0) + H(\gamma, \varepsilon_w)\,\xi + \ldots, \tag{47}$$

where

$$H(\gamma, \varepsilon_w) = -\frac{(\gamma^4 - 1)}{(\gamma - 1)}\{\Theta_1'(0) + (\varepsilon_w - 1)\,\Theta_2'(0)\}. \tag{48}$$

The first term on the right side of equation (47) is the convective heat transfer in the absence of radiation, while the second term is the effect of radiation in a first approximation.

Method of Solution

The velocity and temperature profiles are given by equations (21)-(29). These equations were solved numerically on a SDS-3600 electronic computer. It should be noted that the momentum equations (21) and (22) do not depend on the energy equation and, hence, can be solved separately. The general method of solution can be described as follows. Since F"(0) is the only unknown quantity in the momentum equations (21) and (22) an initial rough estimate of F"(0) is made for some prescribed value of $\eta_{v\delta}$ and the momentum equation is then integrated with all the boundary conditions taken into account. On solving the momentum equation we obtain a function $F''(0)$, which is then used to solve the heat conduction equations (25)-(29). These equations are linear, fortunately, and the solution is obtained in the following way.

We assume that Θ_0 has the form

$$\Theta_0 = h + \Theta_0'(0)\, g, \tag{49}$$

where h and g satisfy the equations

$$\frac{1}{\mathrm{Pr}} h'' + Fh' = 0, \tag{50}$$

$$\frac{1}{\mathrm{Pr}} g'' + Fg' = 0 \tag{51}$$

with boundary conditions

$$h(0) = 1, \quad h'(0) = 0; \tag{52}$$

$$g(0) = 0, \quad g'(0) = 1. \tag{53}$$

We can now determine $\Theta_0'(0)$ from the condition $\Theta_0(\eta_{v\delta}) = 0$ in the form

$$\Theta_0'(0) = -\frac{h(\eta_{v\delta})}{g(\eta_{v\delta})}\,. \tag{54}$$

A similar method is used to solve equations (26) and (27).

Conclusions and Discussion

The boundary-layer differential equations for forced-convection film boiling were solved for various values of $\eta_{v\delta}$, Pr, $((\rho\mu)_v/(\rho\mu)_L)^{1/2}$ and γ.

The results of interest are given in Table 1 and are shown graphically in Figs. 2-9 for $\eta_{v\delta} = 1\text{-}2$, $\gamma = 1.3\text{-}4$; $(\rho\mu) = 0.01$, and $\mathrm{Pr} = 1.0$. Figure 2 shows that the presence of radiation increases the vapor temperature. Hence, heat condition is increased on the vapor–liquid interface, but is reduced on the wall surface.

Figure 3 shows the dimensionless temperature gradients on the wall and interface corresponding to a zero-order perturbation. These gradients are identical when the radiation of the vapor is neglected. As the figure shows, the difference between the temperature gradients on the wall and on the interface is small for thin vapor films (small $\eta_{v\delta}$) and large for thick films (large $\eta_{v\delta}$). This can be attributed to the fact that energy transfer by convection is not important for thin vapor films, but is very important for thick vapor films.

Figure 6 shows function $F(\eta_{v\delta})$ for $(\rho\mu) = 0.01$ and 0.1; $F(\eta_{v\delta})$ is the dimensionless vaporization rate. The figure shows that the vaporization rate increases as the film thickness increases. The figure also shows that $(\rho\mu)$ has an insignificant effect on $F(\eta_{v\delta})$.

It should be noted that the curves of $F(\eta_{v\delta})$ for values of $(\rho\mu)$ equal to 0.01 and 0.001 almost coincide. Without radiation of the vapor, $F(\eta_{v\delta})/[-\Theta_0'(\eta_{v\delta})]$ is equal to the parameter $c_p\Delta T/h_{fg}\mathrm{Pr}$. This quantity is shown as a function of $\eta_{v\delta}$ in Fig. 6. This quantity increases with increase in the vapor film thickness.

Tables 1-3 and Figs. 3-8 give all the information required for calculation of the heat transfer in forced-convection film boiling. For any prescribed boiling conditions (T_w, T_{sat}, liquid, U_∞) the physical properties can be found from the tables, and the physical parameters [$c_p\Delta T/h_{fg}\mathrm{Pr}$, $(\rho\mu)$, and so on] are determined from Table 1 and Figs. 3-8; the dimensionless vapor film thickness $\eta_{v\delta}$ can be determined from equation (35) by trial and error. A value of $\eta_{v\delta}$ is assumed and then the values of $\Theta_0'(\eta_{v\delta})$, $\Theta_1'(\eta_{v\delta})$, $\Theta_2'(\eta_{v\delta})$, I, and $F(\eta_{v\delta})$ are taken from Figs. 3, 5, 7, and 6, respectively. All these values are then substituted in (35). If (35) is not satisfied another value of $\eta_{v\delta}$ must be taken and the calculations repeated until the required value of $\eta_{v\delta}$ is found. Finally, the heat transfer results can be calculated from equation (44). It should be noted that, as Fig. 8 shows, $G(\gamma)$ is always positive. Hence, the effect of the first-order term in equation (41) must reduce the heat transfer relative in the heat transfer given by equation (45). Figure 9 shows $[(\gamma-1)/(\gamma^4-1)]\, H(\gamma, \varepsilon_w)$ as a function of γ for different values of ε.

NOTATION

a_λ	is the monochromatic absorption coefficient;
a	is the absorption coefficient;
c_p	is the specific heat at constant pressure;
C_1, C_2, C_3	are the dimensionless parameters, equation (36);
e_λ	is the monochromatic emissivity of blackbody;
$E_n(t)$	is the exponential integral;
F	is the dimensionless vapor stream function, equation (19);
f	is the dimensionless liquid stream function, equation (19);
$G(\gamma)$	is the function given by equation (42);
h_{fg}	is the heat of vaporization;
$H(\gamma, \varepsilon_w)$	is the function given by equation (48);
I	is the integral given by equation (36);
k	is the thermal conductivity;
L	is the relative length;
m	is the vaporization rate per unit area;
Nu	is the convective Nusselt number;
Pr	is the Prandtl number, $Pr = \mu c_p/k$;
q	is the local heat transfer rate per unit area;
Re	is the Reynolds number;
t	is the variable of integration;
T	is the temperature;
$\Delta T = T_w - T_{sat}$	
u	is the velocity component in x direction;
U_∞	is the free-stream velocity;
v	is the velocity component in y direction;
x	is the coordinate measured along plate from leading edge;
y	is the coordinate perpendicular to plate;
α	is the thermal diffusivity;
γ	is the temperature ratio, T_w/T_{sat};
δ	is the thickness of vapor film;
ε	is the emissivity;
$\zeta = ax/\sqrt{Re}$;	
η	is the similarity variable, equation (19);
$\eta_{u\delta}$	is the dimensionless thickness of vapor film;
Θ	is the dimensionless temperature;
λ	is the wavelength;
μ	is the absolute viscosity;
ν	is the kinematic viscosity;
ρ	is the density;
σ	is the Boltzmann constant;
τ_λ	is the monochromatic optical thickness;
τ	is the optical thickness;
$\tau_{\delta\lambda}$	is the optical thickness of vapor film;
ψ	is the stream function.

Subscripts

c	conduction from wall to vapor;
L	liquid;
l	quantity determined at relative length;
r	radiation;
sat	saturated;
w	wall;
δ	on liquid–vapor interface;
λ	monochromatic property. Dash denotes differentiation with respect to η.

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